# 13 A STOCHASTIC FEEDBACK MODEL FOR OPTIMAL MANAGEMENT OF NAMIBIAN SARDINE

*Stein Ivar Steinshamn, Arne-Christian Lund and Leif K. Sandal*[*]

*Abstract*

A stochastic feedback model is applied in order to derive optimal harvesting strategies for the Namibian sardine that takes into account socio-economic considerations in addition to biological safety and sustainability. There are certain features pertaining to the sardine fishery that need special attention. These features are the technology of the fishery, uncertainty and stochasticity and market conditions. One of the main results from this study is that more uncertainty does not necessarily imply more conservative management and lower harvest; under certain circumstances the opposite may be the case. Based on what we think is the most realistic version of the model, the main result is that in the present situation, that is for stock levels less than about 500 000 tonnes, a harvest moratorium should be instituted. The optimal harvest path is then a piecewise curve due to the demand function. The optimal long-term steady state is a stock at around two million tonnes, and the economic optimal harvest is around 450 000 tonnes at this stock level. The overall conclusion, then, is that in the present situation severe measures have to be taken in order to get the sardine stock out of the trap where it is at present. If the right measures are taken, the sardine stock may give a valuable contribution to the Namibian economy in the future, otherwise it may disappear for a long time. However, 2002 is the first year since 1981 with a zero TAC, indicating that drastic action is now beginning to take place.

## INTRODUCTION

In this chapter a stochastic model is applied in order to derive optimal harvesting strategies for the Namibian sardine that takes into account socio-economic considerations in addition to biological safety and sustainability. The deterministic version of the model is described by Sandal and Stein-

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shamn (1997a, 2001a) and the stochastic version by Sandal and Steinshamn (1997b). Application of the deterministic model to the Arcto-Norwegian cod and Atlanto-Scandian herring can be found in Arnason *et al.* (2000), to the cod off Newfoundland in Grafton *et al.* (2000) and to Namibian sardine in Sandal and Steinshamn (2001b). Application of the stochastic model to Southern Bluefin Tuna can be found in McDonald *et al.* (2002).

There are certain features pertaining to the sardine fishery that need special attention. These features are the technology of the fishery, uncertainty and stochasticity and market conditions. More about the sardine can be found in other chapters in this book, for example the chapters by Boyer and Oelofsen or Sumaila and Steinshamn.

The Namibian sardine fishery is characterized by purse seine technology. This means that there is relatively little relationship between stock size and harvesting costs. The costs of catching one school of fish do not depend on the total number of schools to any large extent. An important implication of this is that the optimal steady state may not be as sensitive to economic parameters as otherwise would have been the case.

The Namibian sardine is also characterized by a high degree of uncertainty and stochasticity in the biological functions. This is mainly due to the unpredictable Benguela Current system, and it calls for explicit inclusion of rather general stochastic processes in the models.

Despite the stochasticity and unpredictability it is possible to detect certain time trends in the productivity of the sardine stock. The biology of the stock seems to go through different regimes over time. This calls for independent analysis of different time periods, and it gives interesting and novel insight regarding the biological productivity of the sardine stock in different periods.

One of the main results from this study is that the consequence of introducing stochasticity is not monotone regarding optimal management. In other words, more uncertainty does not necessarily imply more conservative management and lower harvest, under certain circumstances the opposite may be the case.

## DETERMINISTIC MODEL[1]

The general dynamic optimization can be described by a couple of equations, namely:

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[1] This section is rather technical and can be skipped by those who are mainly interested in the results.

$$W = \int_0^\infty \Pi(x, y, t)dt \tag{1}$$

subject to the dynamic constraint

$$\dot{x} = g(x) - y$$

and the appropriate transversality conditions. Here $x$ is the fish stock, $y$ denotes the yield rate (or harvest rate) from the stock and $t$ denotes time. The function $g(x)$ is the biological surplus production function for the stock. Time is the basic variable here as the variables $x = x(t)$ and $y = y(t)$ are themselves functions of time. Dot-notation is used to represent time derivatives, that is:

$$\dot{x} \equiv \frac{dx}{dt}.$$

The function $W$ is the objective that is to be maximized. Further, the function II can represent private net revenue, that is producers' surplus, or some measure of social welfare, for example the sum of producers' surplus or consumers' surplus. In this chapter, II will usually represent the producers' surplus for the harvesting sector, and it assumed that II is concave. The processing sector is also important for the Namibian economy, but due to lack of economic data this sector has to be excluded from the analysis.

In the basic version of the model dealt with here the only sort of explicit time dependence will be discounting of the future. That is,

$$\Pi(x, y, t) = e^{-\delta t}\Pi(x, y)$$

where δ is the discount rate (and $e^{-\delta t}$ is the discount factor). The method applied to solve this maximization problem is optimal control theory, see, for example, Kamien and Schwartz (1991) or Clark (1990) for application to fisheries.

In the following all variables are in current values. The Hamiltonian for the problem is given by

$$H(x, y, m) = \Pi(x, y) + m \cdot [g(x) - y] \tag{2}$$

where $m$ is the so-called costate variable. The first-order conditions for maximum are[2]

$$H_y = 0, \quad \dot{x} = g(x) - y$$
$$\dot{m} = \delta m - H_x, \; \dot{H} = \delta m \dot{x}.$$

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2 Subscripts are used to denote partial derivatives.

The last of these equations follows from the three previous ones. The conventional approach in optimal control theory would be to find an alternative expression for $m$ from $H_y = 0$, equate this to the expression we already have for $m$, and use this together with the equation for $x$ to construct a system of two differential equations, one for $x$ and one for $y$. A typical approach is to set these equal to zero and solve the system for the steady state.

In this chapter we are more interested in the optimal way to approach steady state than we are in the steady state itself. The reason for this is that faced with practical management problems one is usually far away from the optimal steady state, and it is therefore not sufficient to know the optimal policy only in, or very close to, the steady state. We therefore take the following alternative approach. The so-called maximum principle, $H_y = 0$, implies that $\Pi_y - m = 0$. From this follows that the costate variable (which in optimum can be interpreted as the shadow value of the resource) can be written as a function

$$m = M(x, y) = \Pi_y.$$

This is a known function when $\Pi$ and $g$ are known, and it can therefore be used to eliminate $m$ from the system. Substituting $M$ for $m$ in the Hamiltonian we get a new function that is equal to the Hamiltonian in value along an optimal trajectory, but different as a function. This new function can be defined as

$$P(x, y) = H(x, y, M(x, y)).$$

This function has the same interpretation as the Hamiltonian, namely as the rate of increase of total assets, and it will here be called total economic rent for short.

Here we are primarily looking for optimal policies to set total allowable catches as a function of the stock level, that is to find $y$ as a function of $x$, $y = y(x)$. This inserted into $H = \delta m x$ from (2) yields a first-order differential equation that can be used to determine the feedback control. This equation can be written

$$\frac{dP}{dx} = \frac{\partial P}{\partial x} + \frac{\partial P}{\partial y}\frac{dy}{dx} = \delta \cdot M(x, y). \tag{3}$$

Equation 3 is the basic equation that will be used in the deterministic model. The term feedback is important here. This means that we have a rule to determine optimal harvest as a function only of the last observed stock estimate given that the parameters in the model have been properly estimated. The implication of this is that management becomes genuinely adaptive, and no forecasting is required.

## BIOLOGICAL MODEL

Biological fisheries models can be divided in two broad categories: aggregated and disaggregated (year-class) models. Year-class models are primarily used for short term purposes, mainly stock assessment. These usually have a high degree of detail with respect to individual weight, sexual maturity, natural mortality, etc., between year-classes, that are useful for short-term prediction of stock abundance. These details are less useful for long-term predictions, however, as such biological characteristics tend to vary over time. Hence aggregated biological models are more useful for describing long-term variations of the stock, and derive long-term harvesting strategies, as they use averaged parameters.

Several functional forms of the surplus production model given by Equation (1) were tried, and the following gave the best statistical properties[3]

$$g(x) = \alpha x^3 + \beta x^4. \tag{4}$$

The function given by (4) entails depensation, but not critical depensation. Depensation means that there is a convex area of the function close to the origin implying that the production of the stock is low here. Critical depensation means that there is a convex area with negative growth close to the origin. In a deterministic model this implies that below a certain stock level the population will go extinct even without any harvest, see also Clark (1990). The apparent collapses of the stock in the mid-sixties, and again in the mid-seventies, as seen from Figure 1 may partly be attributed to such depensation.

The results of the estimation are given in Table 1 with biomass measured in 1000 tonnes.

Table 1. Statistical properties of the surplus production model estimated with data from 1969 to 2000 (1975 removed as an outlier).

| Parameter | t-value | |
|---|---|---|
| $\alpha$ = 3.87E-7 | 6.7 | $R^2$ = 0.60 |
| $\beta$ = -1.61E-10 | -6.2 | F = 45.3, DW = 1.21 |

## ECONOMIC MODEL

The main components of the economic model are revenue and costs. As the

[3] The estimations were performed using the program NLREG.

objective of the model is to maximize the welfare of the Namibian people, it would be natural to include the harvesting as well as the processing sector; that is, to maximize the producers' surplus from both these sectors. Unfortunately it has not been possible to get sufficient data from the processing sector to include this in the optimization, so we concentrate entirely on the harvesting sector in the following. The so-called consumers' surplus is not included in the welfare concept, as most of the fish products based on sardine are exported abroad, mainly to South Africa.

**Gross Revenue Function**

The revenue function can be described by the equation

$$R(y) = p(y)y$$

where $y$ is harvest as earlier and $p(y)$ is the inverse demand function. The inverse demand is the price as a function of output instead of the other way around. In general the demand is a function of several variables, in particular own price, price of close substitutes and income. As income and price of substitutes typically change over time, this would call for non-autonomous, or time-dependent, functions. Here we concentrate on demand as a function of own price, or rather the inverse, the price as a function of output or harvest. For simplicity we drop the word inverse in the following.

We do not attempt to estimate the demand as a function of multiple variables. This is partly because sufficiently good data do not exist and partly because it is not required for this type of model. What is needed here is to find a formulation of the revenue function that can describe reality reasonably well.

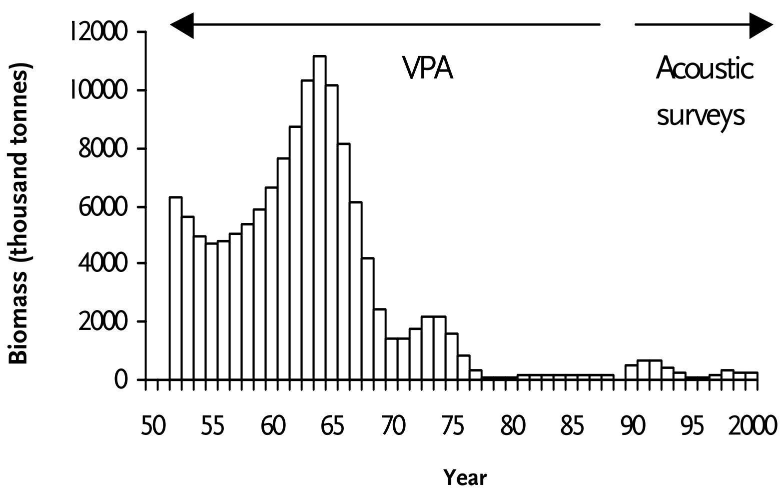


Figure 1. Estimated biomass of Namibian sardine. Source: VPA estimates are from Butterworth (1983). Acoustic estimates are annual means.

In this work the following function has been applied:

$$p(y) = \frac{ap_{max} + p_{min} y}{a + y} \tag{5}$$

where $p_{max}$ is the maximum price, $p_{min}$ is the minimum price and $a$ is a parameter to be determined. This function has some useful characteristics: When $y = 0$ the price is $p_{max}$ and when the quantity approaches infinity ($y \to \infty$) the price approaches $p_{min}$. The maximum and minimum price can be pre-specified or estimated from data. Given that the parameter $a>0$, the demand function is always downward sloping

$$(\frac{dp}{dy} < 0)$$

and convex

$$(\frac{d^2 p}{dy^2} > 0).$$

Furthermore, this function is also always elastic. The elasticity of demand as a function of $p$ is given by

$$El = \frac{p(p_{max} - p_{min})}{(p_{max} - p)(p - p_{min})}.$$

It is relatively easy to show that this is always greater than one, and it is at its minimum when $p = \sqrt{p_{max} * p_{min}}$. Further, $El \to \infty$ both when $p \to p_{max}$ and $p \to p_{min}$. With this demand function, gross revenue will be an increasing but concave function in output.

**Cost function**

The cost function applied here is supposed to include the costs of the harvesting sector. The costs are in general supposed to be a function of both the stock size and the harvest. In the special case of the sardine fishery, these costs, however, are only weakly dependent upon the size of the fish stock. The reason for this is that the sardine is a schooling species and the fishery is mainly purse seining. It is usually almost as easy to locate and catch a school of fish when the stock is large as when the stock is small. Another way to say this is there is relatively little relationship between the stock size and catch per unit effort. Hence there is little reason to believe that costs will go down when the stock increases for the same quantity harvested. To the extent that there is any difference, it is the time it takes to locate the schools that differs when the total stock changes.

On the other hand, it is natural to assume that the costs are convex in the harvested level. One functional form that can be relevant for this case is

$$c(x,y) = k\frac{y^2}{x^b} \tag{6}$$

where $k$ and $b$ are non-negative parameters, and $b$ is small. This includes both the convexity with respect to harvest and the weak dependence upon the stock. If $b = 0$, there will be no relationship between costs and the stock level. On the other hand, if $b \geq 1$, there will be a strong relationship, which is typical for demersal fisheries. A $b > 0$ also provides an economic guarantee against extinction as the cost of harvesting goes to infinity when the stock approaches zero. Further the cost function has the following characteristics:

$$c_y > 0,\ c_{yy} > 0,\ c_x \leq 0.$$

This means that costs are increasing and convex in harvest and decreasing in the stock.

**Net revenue function**

The net revenue function is simply gross revenue minus costs:

$$\Pi(x,y) = R(y) - c(x,y) = p(y)y - c(x,y).$$

As we have seen, this function can in principle be completely general in both arguments. With the functions given above, we are guaranteed that the net revenue function is meaningful, and that the sufficiency conditions are fulfilled.

## ESTIMATING THE ECONOMIC SUBMODEL

It is much harder to find relevant economic data for sardine than relevant biological data. Cost data are especially difficult to obtain as is the case in most fisheries. The two main uses of sardine are either canning or fishmeal, and the price for fish going to canning has always been higher than for fish going to fishmeal. This price difference has increased over the 90s. Whereas the price for canning has increased steadily the price for meal has been stable for long periods.

The volume going to meal has been decreasing; also in periods when total catch increased. Note, however, that in the entire period 1980 - 1999 the catch has not been above 120 000 tonnes. For larger quantities harvested a larger proportion will be used for meal.

**Estimating the demand function**

In order to determine a reasonable demand function for sardine, one has to take into account that the harvest is partly used for canning, and partly used for fishmeal. For small quantities most of the catch goes to canning whereas for larger quantities it is expected that most of the catch goes to fishmeal. In the following some simplifying, but reasonable, assumptions are made: for TACs less than 60 000 tonnes everything goes to canning, for the part of the TAC between 60 and 120 000 tonnes 80 per cent goes to canning and 20 per cent to fishmeal, and for the part of the TAC over 120 000 tonnes everything exceeding 108 000 tonnes goes to fishmeal. Letting *y*1 denote canning and *y*2 fishmeal, the following relationship appears:

$$y1(y) = \begin{cases} y & if \quad y < 60 \\ 12 + 0.8y & if \quad 60 < y < 120 \\ 108 & if \quad y > 120 \end{cases} \tag{7}$$

$$y2(y) = \begin{cases} 0 & if \quad y < 60 \\ -12 + 0.2y & if \quad 60 < y < 120 \\ -108 + y & if \quad y > 120 \end{cases} \tag{8}$$

Over 90% of the canned sardine is exported to South Africa, but there is a limit to how much the South African market can consume, and this depends on the South African harvest of sardine. Over time the limit varies between 60 000 and 120 000 tonnes, and for the time being it is believed to be close to 60 000 tonnes (personal communication with Les Clark at the Ministry of Fisheries and Marine Resources, Windhoek). This is reflected in the function above. The distribution between canning and meal described here is only relevant for the period from the 1990s and onwards.

Figures 2 and 3 show observed prices and quantities for canned sardine and fishmeal. The year 1996 has been removed as total harvest this year was very small. In addition 1994 was removed for canning and 1995 for meal. These observations were removed after visual inspection of the data before running the regressions. An explanation why 1995 was an outlier may be that about half of the fish was caught in Angolan waters, and, due to the distance, this caused low quality and a low ratio of canning to meal.

The prices have been converted to fixed 1998-prices. These observed prices are used to estimate functions of the form in (5). The data for fishmeal give better estimates than for canning. For fishmeal we have estimated both the minimum price and the maximum price as well as the parameter *a*. For canning the maximum price has been pre-determined to be 2.2 N$/kg (per-

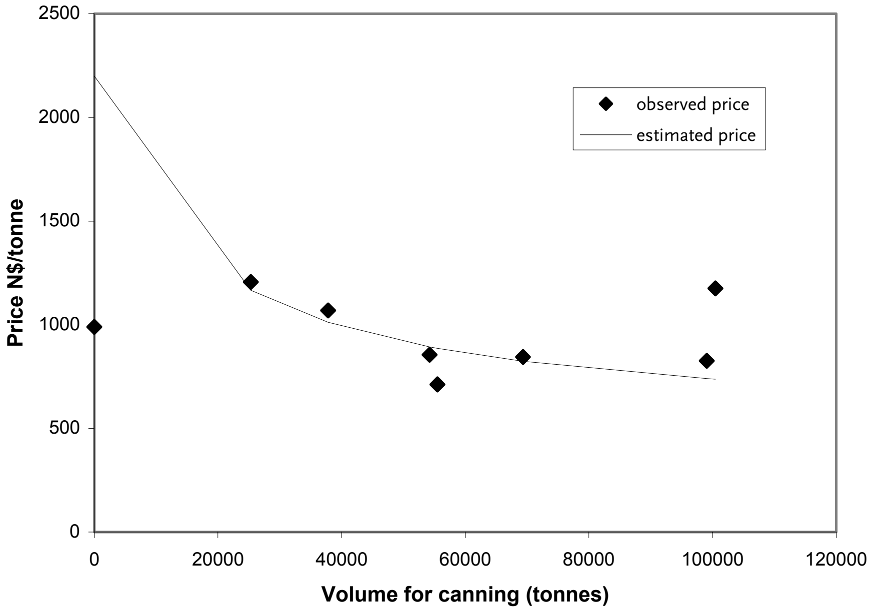


Figure 2. Observed and estimated prices for canning against volume from 1990 to 1997 (1994 and 1996 are removed as outliers).

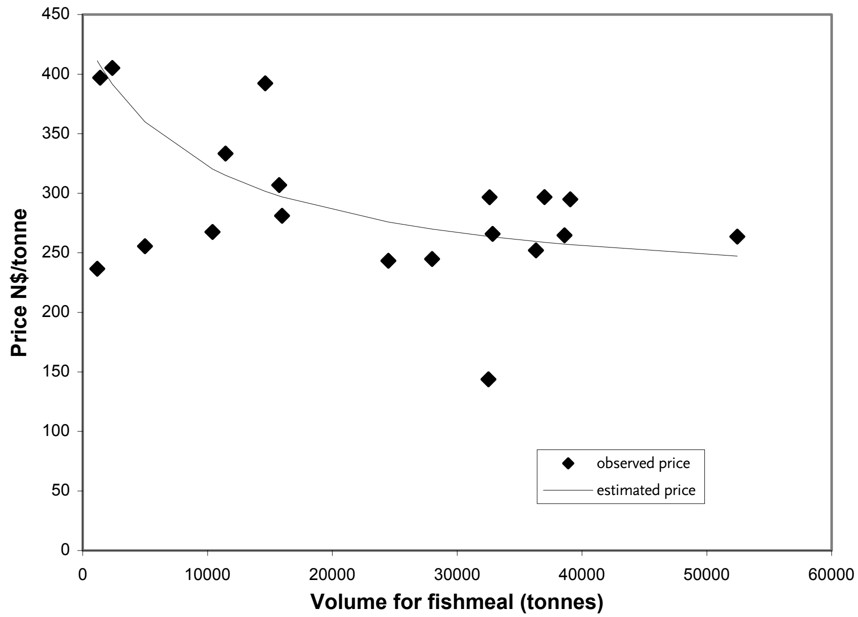


Figure 3. Observed and estimated prices for meal against volume from 1980 to 1998 (1995 and 1996 are removed as outliers).

sonal communication with Les Clark at the Ministry of Fisheries and Marine Resources, Windhoek), and the minimum price and parameter $a$ have been estimated. The statistical properties of the estimations are given in Tables (2) and (3).

With the values from these tables we have defined the price of canned sardine as a function of the volume of canned sardine,

$$p1(y1) = \frac{16.3 \cdot 2.2 + y1 \cdot 0.5}{16.3 + y1},$$

and the price of fishmeal as a function of the volume going to fishmeal,

$$p2(y2) = \frac{9.8 \cdot 0.44 + 0.21 \cdot y2}{9.8 + y2}$$

However, $y1$ and $y2$ are functions of the total allowable catch, $y$, given in (7) and (8), and therefore $p1$ and $p2$ are functions in $y$ only, too. Hence the average price defined as total value divided by total volume is given by

$$p(y) = \frac{p1(y1)y1(y) + p2(y2)y2(y)}{y}. \tag{9}$$

This function is a piecewise, non-linear function as depicted in Figure 4.

**Calibrating the cost function**

The cost function is calibrated on the assumption that there is a break-even point where gross revenue meets the cost function. We have assumed that this break-even point is at a harvest level in the region 1.1 to 1.4 million tonnes. This assumption is based on observations of the fishery in the mid-

Table 2. Statistical properties of the estimation of the demand for canned sardine.

| Parameter | Value | t-value | |
|---|---|---|---|
| $A$ | 16.3 | 2.24 | $R^2 = 0.74$ |
| $p_{min}$ | 0.5 | 2.69 | F = 11.1 |

Table 3. Statistical properties of the estimation of demand for fishmeal.

| Parameter | Value | t-value | |
|---|---|---|---|
| $A$ | 9.83 | 0.78 | $R^2 = 0.52$ |
| $p_{max}$ | 0.435 | 6.7 | DW = 1.4 |
| $p_{min}$ | 0.212 | 3.52 | F = 7.45 |

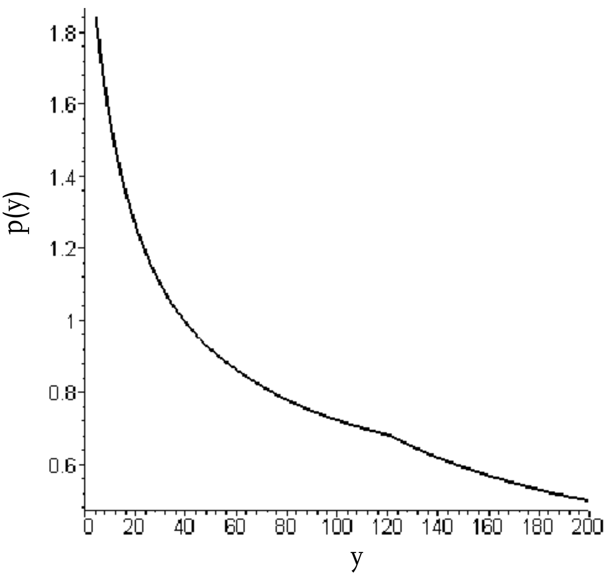


Figure 4. Illustration of p(y).

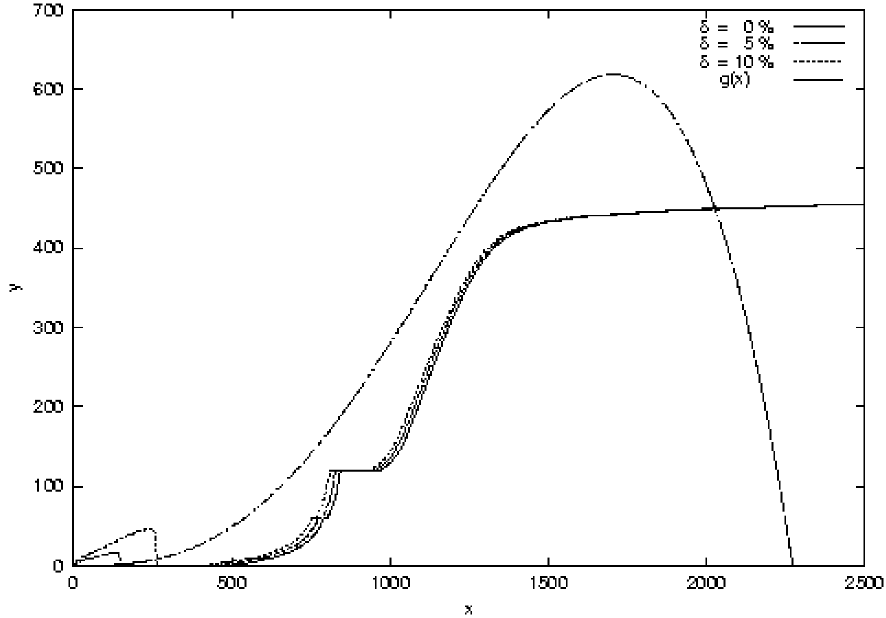


Figure 5. Optimal harvest paths from the deterministic model with discount rates of 0%, 5% and 10% together with the surplus production.

sixties when the system, for all practical purposes, can be treated as open access although some regulations (licenses, TAC) were in place. According to the theory, open access leads to exhaustion of the resource rent. With this assumption the value of the parameter $k$ in (6) is 0.00034. The value of $b$ is a pure guesstimate and is set to 0.05. In other words, the specification of the cost function is:

$$c(x,y) = 0.00034\frac{y^2}{x^{0.05}}. \tag{10}$$

The weak part of the economic model is the cost analysis. Sensitivity analysis with respect to the cost parameter on this model was performed by Sandal and Steinshamn (1999). There it was found that a 50% increase in the cost parameter $k$ implies 5% increase in the optimal steady state stock and 33% decrease in optimal steady state harvest whereas a 50% decrease in the cost parameters implies 15% decrease in the stock and 36% increase in harvest.

## THE EMPIRICAL MODEL

The empirical model applied here is the one where the demand function is given by Equation (9) and illustrated in Figure 4. The cost function is given by (10), and the biological model is the surplus production model based on data from 1969 - 2000:

$$g(x) = 3.9E-7\cdot x^3 - 1.6E-10\cdot x^4. \tag{11}$$

Now stock and harvest are measured in 1000 tonnes instead of millions. The optimal harvest paths with these assumptions and different discount rates are illustrated in Figure 5. The case with zero discounting is of special interest as sardine is a renewable resource, and therefore we want to put as much emphasis on the future as on the present. We can see, however, that optimal harvest paths are not very sensitive to changes in the discount rate. This is always the case as long as the discount rate is small relative to the intrinsic growth rate of the stock. A detailed discussion of the deterministic model can be found in Sandal and Steinshamn (1999 and 2001b).

Note that for stock levels below 500 000 tonnes a harvest moratorium is recommended. It is interesting to note that this is very close to the level proposed by Fossen *et al.* (2001) although their recommendation is based solely on biological considerations. Even if the implications on employment of a harvest moratorium are taken into account, the moratorium is still optimal because the long-term effects by far outweigh the short-term effects. In other words, to increase employment temporarily by having higher quotas than the optimal ones will severely decrease the possibility of having a high employment in the future due to the adverse stock effects. As we shall see, this effect is reinforced when stochasticity is introduced.

**Changing the discount rate**

So far the study of optimal harvest paths has been performed with zero discounting. Optimal paths, however, are not very sensitive to changes in the discount rate as long as the discount rate is small relative to the intrinsic growth rate of the stock. This has been shown both theoretically and in practice (Sandal and Steinshamn, 1997a and c). It is further illustrated for Namibian sardine in Figure 5 where optimal harvest paths are drawn for a discount rate of 0%, 5% and 10% based on the same basic model as earlier.

With zero discounting it is optimal to implement a moratorium on harvest for stock levels below a certain limit. When the discount rate is increased to 5% and 10% we observe positive harvest for small stock levels followed by a small region of harvest moratorium. As the discount rate increases there will be competition between the discount rate and the intrinsic growth rate of the stock. This means that it may pay off to increase harvest in order to put the money in the bank instead of letting the fish grow in the ocean, for very small stock sizes. This is no problem as long as the optimal harvest is below the growth curve in a deterministic model because the stock will increase towards the optimal steady state anyway, although slower than with a smaller discount rate. If the stock level is so small that the harvest curve is above the growth curve, on the other hand, it is economically optimal to wipe out the rest of the resource according to the model. This is, of course, not an acceptable strategy in practical management. In practice it is probably not even possible as it is usually very difficult to find a statistically significant relationship between spawning stock biomass and recruitment for small pelagic fish. An ideal model should also include functions that represent the social and political costs of extinction. The validity of the biological model is limited close to the origin, and one should be careful, especially when high discount rates are used. For most values of the stock, optimal harvest is quite insensitive to reasonable changes in the discount rate. For small stock sizes, however, a change in the discount rate may imply the difference between the existence of a moratorium stock level or not. In other words, it is not the optimal harvest that is sensitive to the discount rate, but the moratorium stock level can be sensitive. Generally, higher discounting implies less conservative harvesting.

## RESULTS FROM THE STOCHASTIC MODEL

So far it has been assumed that the biological growth is a deterministic process within the various periods we look at. It is, however, a well-known fact that the Benguela upwelling system is a highly uncertain and irregular system. This calls for inclusion of stochastic processes in the optimisation models. The interesting question is whether such processes have major impacts on the optimal paths and in what direction the paths are affected. In other words, what are the quantitative and qualitative implications of including stochasticity? This question will be answered both in the case of zero discounting and with positive discounting, but first a short description of the stochastic model will be given.

## THE STOCHASTIC MODEL

In the stochastic case we want to maximize expected returns defined as

$$\max_{y \geq 0} E \int_0^{\infty} e^{-\delta t} \Pi(x, y) dt$$

where $E$ is the expectation operator. The dynamics of the stock $x$ is given by

$$dx = \left[ g(x) - y \right] dt + \sigma(x) dw$$

where $\sigma(x)$ represents the stochastic term (or the volatility function) and $dw$ is a standard Wiener process increment. The term $g(x) - y$ represents the deterministic part of the dynamic equation. The volatility function, $\sigma(x)$, can be any function of $x$ in principle. Here the conventional approach of linear functions is used; that is, the uncertainty is increasing proportionally with the stock size.

Dynamic programming is used to solve this problem, and technical details on the numerical solution of such problems can be found in Kushner and Dupuis (2001). Here we only concentrate on numerical solutions of specific cases and not on analytical and theoretical aspects of the problem.

## RESULTS FROM THE STOCHASTIC MODEL WITHOUT DISCOUNTING

The results from the stochastic optimization with three different stochastic processes are illustrated in Figure 6. The stochastic process is given as

$$\sigma(x) = \sigma_0 x,$$

and the three values of $\sigma_0$ that have been applied here are $\sigma_0 = 0.15$, $\sigma_0 = 0.3$ and $\sigma_0 = 0.45$. It is difficult to estimate the parameter $\sigma_0$ from data, but there

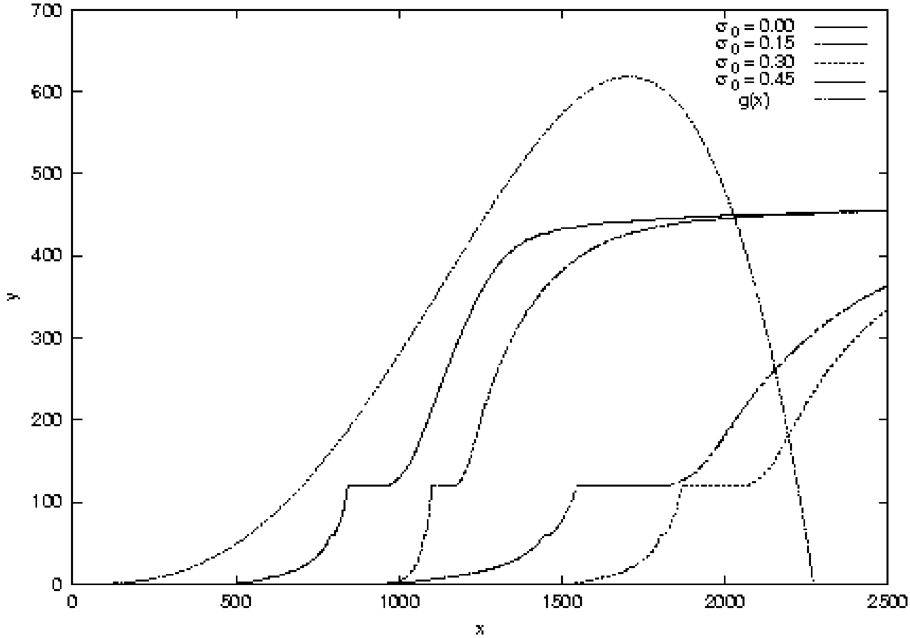


Figure 6. The deterministic optimal path and three stochastic optimal paths together with the surplus production when the discount rate is zero.

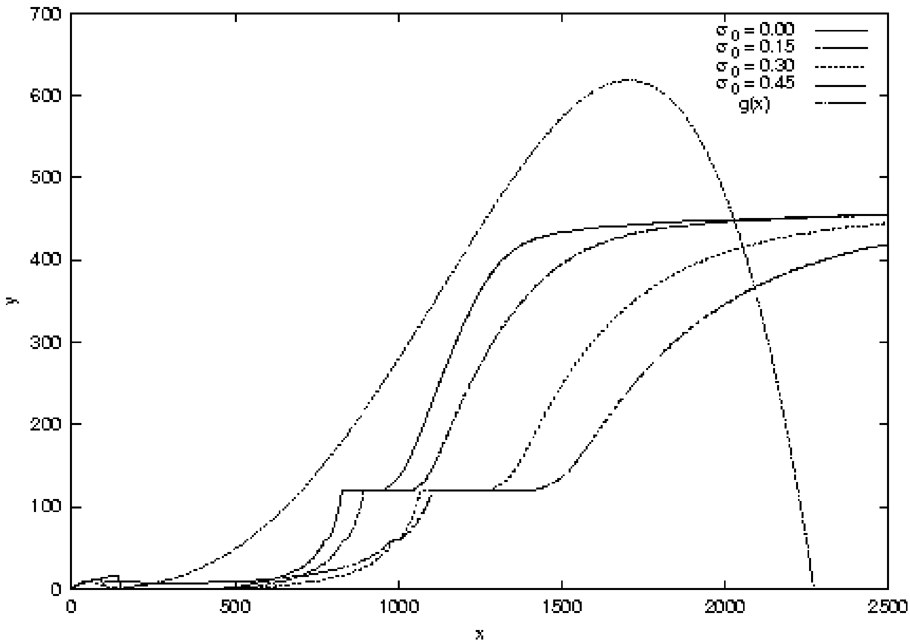


Figure 7. The deterministic optimal path and three stochastic optimal paths together with the surplus production when the discount rate is 5%.

are indications that the noise in this system is large. An indication that $\sigma_0$ is reasonable is that the standard deviation of $g$ and $\sigma_0 \cdot x$ for a representative $x$-value are in the same order of magnitude. The standard deviation of $g$ from the data is 213 whereas a typical $x$-value is half of the carrying capacity, namely 1200. Thus all three values of $\sigma_0$ suggested above are reasonable.

With more data more sophisticated functional forms of $\sigma$ than the simple linear one could have been applied. The ideal situation would have been to have replicated time series such that $g$ and $\sigma$ could have been estimated simultaneously (see McDonald and Sandal, 1999). In the case when $g$ and $\sigma$ are estimated simultaneously, we get a different $g$ from the one obtained when $g$ is estimated in a purely deterministic context.

There are several interesting points to note from Figure 6. First we observe that the optimal control curves are constant at certain levels. This effect is due to the piecewise definition of the demand function. We also observe that the two lowest values of $\sigma_0$ imply a more careful policy than the deterministic model. They also introduce harvest moratorium on substantially higher stock levels. When the noise parameter is increased from 0.30 to 0.45, however, optimal harvest increases. This indicates that there exists a value of $\sigma_0$ between 0.3 and 0.45 that yields the most conservative policy. Finally we note that for very high stock levels, much higher than the carrying capacity, the three stochastic control curves and the deterministic policy all approach the same level. This level is the harvest level that maximizes net revenue as a function of $x$. In other words, the harvest defined by

$$\frac{\partial \Pi(x,y)}{\partial y} = 0.$$

## RESULTS FROM THE STOCHASTIC MODEL WITH DISCOUNTING

In the description of the empirical model it was seen that in the deterministic case, optimal paths are not very sensitive to changes in the discount rate. In the stochastic case, however, this is no longer necessarily so. The optimal paths for three different values of $\sigma_0$, when the discount rate is five per cent, are illustrated in Figure 7.

We observe positive harvest in the region close to zero, and the stock may be driven to extinction. When $\sigma_0$ is small we also note that the moratorium level is reduced considerably. For large values of $\sigma_0$ there is no moratorium level at all except zero. The main reason for this is high volatility combined with a biological growth function with depensation. It can be shown that the stochastic process implies self extinction almost surely when $\sigma_0$ is large. We may say that this is an "induced critical depensation" process. By induced critical depensation is meant a stochastic model *without* critical depensation that behaves like a deterministic model *with* critical depensation. Therefore it is economically optimal to mine out the rest of the resource in this case and invest the money as it will go extinct anyway. For moral and political reasons this is, of course, not an acceptable possibility in practice. For small $\sigma_0$ levels the self extinction probability is less than one, depending on the initial stock level. When the initial stock is small, the self extinction probability may be large even for $\sigma_0$ close to zero. Again this is due to the low growth rate in this region. The positive self extinction probabilities may also explain the substantial shifts in the moratorium level when stochasticity is introduced. It is now very important to prevent accidental low stock levels where the stock can wipe itself out even with zero harvest.

In addition, there is discounting and therefore there is a marginal trade-off between leaving the fish in the ocean and let it grow or taking it up, selling it and putting the money in the bank. At the margin these two alternatives should be equal. A well-known result from this trade-off is that it may pay off to harvest even the last fish and put the money in the bank if the intrinsic growth rate of the resource is much smaller than the discount rate. Remember that in this context the discount rate is interpreted as the alternative rate of return defined as the rate of return on the best alternative investment.

Usually it is argued that stochasticity should be taken into account when optimizing in order to avoid risk. In a strict dynamic optimisation framework the result may be, as we have seen, that it is optimal to wipe out the whole resource because there is a certain probability that the resource may go extinct anyway. This probability is of course zero in the deterministic version of the model (unless one is faced with critical depensation, see Clark (1990)). On the other hand it is widely agreed that extinction is not a realistic option both for political and for moral reasons.

What is there to be learned, then, from this exercise? The most important result is that introducing uncertainty implied more careful exploitation of the resource, but only for modest noise levels. With high levels we may sometimes get induced critical depensation if the surplus production function has depensation, and the optimal controls change accordingly. They may still be correct if we really believe that the stock faces self extinction. In addition the study confirms the well-known result that discounting is another factor that drives towards less conservative management.

## SUMMARY

The aim of this study has been to find optimal harvest (total allowable catch

strategies) for Namibian sardine when socio-economic as well biological conditions are taken into account. A bio-economic model has been developed for this purpose and the method used is dynamic optimization. In bio-economic models of the kind used here, only long-term, aggregated biological models can be used. The detailed biological models used currently appear to need some improvements because they are based on short time horizons, and therefore the parameter values in them are only relevant for a limited time. In bio-economic models we need average parameter values, valid in the long run. Therefore some effort was spent in the first part of the chapter in order to develop a suitable biological model. Although there are indications that a collapse took place in the mid seventies, data from 1969 to 1994 can be used to estimate a rather good relationship between stock size and growth.

The apparent collapse in the mid seventies may have been due to depensation in the growth function more than to a new biological regime. The presence of depensation, which is supported by the data, may explain why the stock has been trapped at low stock levels since then. We find that a moratorium on harvest is recommendable in order to bring the sardine stock out of the biological trap.

The economic submodel of the bio-economic model consists of two parts: revenue analysis and cost analysis. The aim of the revenue analysis has been to estimate a reasonable demand function for sardine. Demand is divided in two: demand for canned sardine and demand for fishmeal. The volume allocated to each use depends on the total volume. For each demand function there is a maximum and a minimum price that has been estimated together with other parameters. This yields a piecewise non-linear demand function.

The outcome of the bio-economic model is optimal harvest paths as a function of the stock size. In other words, it is an operational model that can be used to determine total allowable catches. Based on what we think is the most realistic version of the model, the main result is that in the present situation, that is for stock levels less than about 500 000 tonnes, a harvest moratorium should be instituted immediately. The optimal harvest path is then a piecewise curve due to the demand function. The optimal long-term steady state is a stock at around two million tonnes, and the economic optimal harvest is around 450 000 tonnes at this stock level. This may, however, be sensitive to changes in the cost function, but under no circumstances should the harvest be higher than the maximum sustainable yield, which is estimated to about 600 000 tonnes. However, before this model could be used for actual management of the stock, a much more thorough empirical analysis of the input functions would have to be made, especially with respect to the cost function.

Applying a stochastic version of the model, it was seen that optimal harvest paths were sensitive to the assumptions about stochasticity as well as the discount rate. With large uncertainty or discounting we find that it may be optimal to wipe out the stock. This is obviously not acceptable for social, political and moral reasons. In future work we will study how political harvest constraints affect the optimal policy.

The overall conclusion, then, is that in the present situation severe measures have to be taken in order to get the sardine stock out of the trap where it is at present. These measures may include a harvest moratorium for several years. If the right measures are taken, the sardine stock may give a valuable contribution to the Namibian economy in the future, otherwise it may disappear for a long time, possibly for ever. However, 2002 is the first year since 1981 with a zero TAC, indicating that drastic action is now beginning to take place.

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